Name: ______________________________

Instructor: __________________________

Math 10550, Practice Exam II
March 4, 2026

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 18 pages of the test.
- Each multiple choice question is worth 7 points. Your score will be the sum of the best 10 scores on the multiple choice questions plus your score on questions 13-15.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice ______________

13. ______________

14. ______________

15. ______________

Total ______________

Name: ______________________________

Instructor: __________________________

Multiple Choice

1.(7 pts.) Find $f''(0)$ if $f(x) = \dfrac{4}{x+2}$.

- $f'(x) = \dfrac{(x+2)\cdot 0 - 4\cdot 1}{(x+2)^2}$
- $= \dfrac{-4}{(x+2)^2}$
- $f''(x) = \dfrac{(x+2)^2\cdot 0 - (-4)\cdot 2(x+2)^1\cdot 1}{(x+2)^4}$
- $= \dfrac{8}{(x+2)^3}$
- $f''(1) = 8/8 = 1$

(a) 1 (b) $\dfrac{1}{2}$ (c) 0 (d) 2 (e) $\dfrac{-1}{2}$

Name: ______________________________

Instructor: __________________________

2.(7 pts.) Let $h(x) = f \circ g(x) - \dfrac{f(x)}{g(x)}$. If $f(3) = 0$, $g(3) = 1$, $f'(3) = 3$, $g'(3) = 4$, $f'(1) = 7$, and $g'(2) = 5$, then find $h'(3)$.

(a) 30 (b) 10 (c) 25 (d) 0 (e) 20

Solution: First, note that for any x, we have $h'(x) = f'(g(x))g'(x) - \dfrac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$. Therefore $h'(3) = f'(g(3))g'(3) - \dfrac{g(3)f'(3) - f(3)g'(3)}{(g(3))^2} = f'(1)g'(3) - \dfrac{g(3)f'(3) - f(3)g'(3)}{(g(3))^2} = 7 * 4 - \dfrac{1 * 3 - 0 * 4}{1^2} = 28 - 3 = 25$

Name: ____________________________

Instructor: _________________________

3.(7 pts.) A fly is moving on a straight line with position at time t is given by

$$s(t) = t\cos(\pi t),$$

where time, t, is measured in seconds and $s(t)$ gives the distance (in feet) of the fly from the origin at time t. What is the acceleration of the fly when $t = 1$?

Solution: $s'(t) = \cos(\pi t) + t(-\pi\sin(\pi t)) = \cos(\pi t) - t(\pi\sin(\pi t))$.

$s''(t) = -\pi\sin(\pi t) - \pi\left[\sin(\pi t) + t\pi\cos(\pi t)\right]$.

$s''(1) = -\pi\sin(\pi) - \pi\left[\sin(\pi) + \pi\cos(\pi)\right] = 0 - \pi\left[0 + \pi(-1)\right] = \pi^2$

(a) $-\pi$ ft/s^2

(b) -1 ft/s^2

(c) 1 ft/s^2

(d) $(-1-\pi)$ ft/s^2

(e) π^2 ft/s^2

Name: ______________________________

Instructor: __________________________

4.(7 pts.) Compute $f'(x)$ if

$$f(x) = \sqrt{1 + \sin^2 x}.$$

$$f'(x) = \frac{1}{2\sqrt{1+\sin^2 x}} \frac{d(1+\sin^2 x)}{dx}$$

$$= \frac{2\sin x}{2\sqrt{1+\sin^2 x}} \frac{d(\sin x)}{dx}$$

$$= \frac{\sin x \cos x}{\sqrt{1+\sin^2 x}}$$

(a) $\sin x \cos x \sqrt{1+\sin^2 x}$

(b) $\dfrac{\sin x \cos x}{\sqrt{1+\sin^2 x}}$

(c) $\dfrac{1}{2\sqrt{2\sin x \cos x}}$

(d) $\dfrac{\sin x}{\sqrt{1+\sin^2 x}}$

(e) $\dfrac{\cos^2 x}{2\sqrt{1+\sin^2 x}}$

Name: ____________________________

Instructor: ________________________

5.(7 pts.) Find y', if

$$x^4 + x^3y + 5xy^2 = 8.$$

(a) $\dfrac{-(4x^3 + 3x^2y + 5y^2)}{10xy}$

(b) $\dfrac{-(4x^3 + 3x^2y)}{x^3 + 10xy}$

(c) $\dfrac{-(4x^3 + 3x^2y + 5y^2)}{x^3 + 10xy}$

(d) The derivative does not exist.

(e) $\dfrac{-(4x^3 + 3x^2y + 5y^2)}{x^3}$

Solution: We differentiate both sides with respect to x

$$\frac{d(x^4 + x^3y + 5xy^2)}{dx} = \frac{d(8)}{dx}$$

$$4x^3 + 3x^2y + x^3\frac{dy}{dx} + 5y^2 + 10xy\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-4x^3 - 3x^2y - 5y^2}{x^3 + 10xy}$$

Name: ______________________________

Instructor: __________________________

6.(7 pts.) If $\sin(\pi xy) = \pi(x+y)$ find $\dfrac{dy}{dx}$ at $(1,-1)$ by implicit differentiation.

Solution: Differentiating both sides with respect to x,

$$\cos(\pi xy)\left(\pi y + \pi x\frac{dy}{dx}\right) = \pi\left(1 + \frac{dy}{dx}\right).$$

Now, letting $x = 1$, $y = -1$ we get

$$\pi\cos(-\pi)\left(-1 + \frac{dy}{dx}\right) = \pi(1 + \frac{dy}{dx}).$$

Isolating dy/dx we get

$$\frac{dy}{dx} = 0.$$

(a) $\dfrac{\pi}{2}$ (b) π (c) 1 (d) 0 (e) -1

Name: ___________________________

Instructor: ________________________

7.(7 pts.) A particle is moving in a straight line along a horizontal axis with a position function given by

$$s(t) = t^2 - 4t + 4,$$

where distance is measured in feet and time is measured in seconds. What is the distance travelled by the particle in the time period $1 \leq t \leq 4$ seconds?

SOLUTION:

Recall that the distance is given by how much has the particle travelled, thus we need to look for any point where our particle changed direction. To do so we consider where (if any) does the derivative of s change signs. We compute

$$s'(t) = 2t - 4$$

Which is zero at $t = 2$ and since it is an increasing line we know $s'(t) < 0$ for $t < 2$ and $s'(t) > 0$ for $t > 2$. Thus it changes direction on $[1, 4]$ only at $t = 2$. Hence the total travelled distance (since the particle is moving in a straight line along a horizontal axis), in feet, is given by

$$D = |s(4) - s(2)| + |s(2) - s(1)| = |4 - 0| + |0 - 1| = 5.$$

(a) 3 feet (b) 0 feet (c) 2 feet

(d) 8 feet (e) 5 feet

Name: ____________________________

Instructor: ________________________

8.(7 pts.) A beetle travels in straight line with position $s(t)$(measured in feet) at time t(measured in seconds), $t \geq 0$. $s(t)$ is given by

$$s(t) = 3t^4 - 20t^3 + 36t^2.$$

At what time, after the motion gets started, does the beetle first come to rest?

Solution: We need to compute the points where $s'(t) = 0$:

$$\begin{aligned} s'(t) = 12t^3 - 60t^2 + 72t &= 0 \\ 12t(t^2 - 5t + 6) &= 0 \end{aligned}$$

The points where $s'(t) = 0$ are $t = 0, 2, 3$, so after the motion starts, the beetle come to rest for the first time at $t = 2$.

(a) This beetle never stops.

(b) $t = 2$

(c) $t = 4$

(d) $t = 1$

(e) $t = 3$

Name: ____________________________

Instructor: _________________________

9.(7 pts.) A right triangle has base x feet and height y feet. If the base increases at 2 ft/second, and the height increases at 1 ft/second, find the rate of change in the area of the right triangle when $x = 8$ and $y = 5$.

SOLUTION:

The formula for the area A of a triangle with base x in feet and height y in feet is given by $A = \frac{1}{2}xy$. Differentiating with respect to t, we get that

$$\frac{dA}{dt} = \frac{1}{2}\frac{dx}{dt}y + \frac{1}{2}x\frac{dy}{dt}$$

Plugging in the values given in the problem, we see that

$$\begin{aligned}\frac{dA}{dt} &= \frac{1}{2}(2)(5) + \frac{1}{2}(8)(1) \\ &= 9\end{aligned}$$

(a) 18 ft^2/second (b) 2 ft^2/second (c) 9 ft^2/second

(d) -1 ft^2/second (e) 10.5 ft^2/second

Name: ______________________________

Instructor: __________________________

10.(7 pts.) Find the derivative of $f(x) = \tan(e^{x^2})$.

$$\frac{d\tan(e^{x^2})}{dx} = \sec^2(e^{x^2})\frac{de^{x^2}}{dx}$$

$$= (\sec^2(e^{x^2})) \cdot e^{x^2}\frac{dx^2}{dx} = (\sec^2(e^{x^2})) \cdot e^{x^2} \cdot 2x$$

$$= 2x\sec^2(e^{x^2})e^{x^2}$$

(a) $\cot(e^{x^2})e^{x^2}$

(b) $-2x\sec^2(e^{x^2})e^{x^2}$

(c) $2x\cot(e^{x^2})e^{x^2}$

(d) $2x\sec^2(e^{x^2})e^{x^2}$

(e) $2x\sec^2(e^{x^2})$

Name: ______________________________

Instructor: __________________________

11.(7 pts.) If 100 grams of radioactive material with a half–life of two days are present at day zero, how many grams are left at day three?

- We have initial amount $m_0 = 100$ and half life $t_{\frac{1}{2}} = 2$ days.
- The amount left after t days is given by $m(t) = m_0 e^{kt} = 100e^{kt}$ for some constant k.
- To find the value of k, we use the fact that the half-life is 2 days. This tells us that $50 = 100e^{2k}$ or $\frac{1}{2} = e^{2k}$. Applying the natural logarithm to both sides, we get $\ln\frac{1}{2} = \ln e^{2k}$ or $-\ln 2 = 2k$.
- Therefore $k = \frac{-\ln 2}{2}$ and $m(t) = 100e^{-\frac{t\ln 2}{2}} = 100(e^{\ln 2})^{-\frac{t}{2}} = 100(2)^{-\frac{t}{2}}$
- After 3 days, we have $m(3) = 100(2)^{-\frac{3}{2}} = \frac{100}{2\sqrt{2}}$ or $\frac{100}{\sqrt{8}}$

(a) 50 (b) $\frac{100}{\sqrt{2}}$ (c) $\frac{100}{\sqrt{8}}$ (d) $\frac{100}{2^{1/3}}$ (e) $\frac{100}{4^{1/3}}$

Name: ____________________________

Instructor: ________________________

12.(7 pts.) Find the derivative of $(x^2+1)^{x^2+1}$.

- We use logarithmic differentiation. Let $y = (x^2+1)^{x^2+1}$. Then

$$\ln y = (x^2+1)\ln(x^2+1).$$

- Differentiating both sides with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(x^2+1)\ln(x^2+1) = 2x\ln(x^2+1) + \frac{2x(x^2+1)}{(x^2+1)} = 2x\big[\ln(x^2+1)+1\big].$$

- Multiplying both sides by y, we get

$$\frac{dy}{dx} = y2x\big[\ln(x^2+1)+1\big] = (x^2+1)^{x^2+1}2x\big[\ln(x^2+1)+1\big]$$

(a) $(x^2+1)^{x^2+1}\big(2x\ln(x^2+1)\big)$

(b) $(x^2+1)^{x^2+1}\ 2x\big(\ln(x^2+1)+1\big)$

(c) $(x^2+1)^{x^2+1}$

(d) $2x(x^2+1)^{x^2}$

(e) This function is not defined and hence has no derivative.

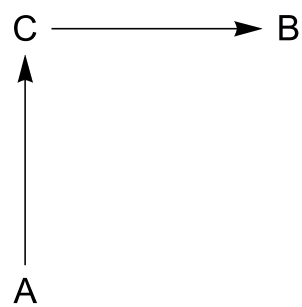
Name: ____________________________

Instructor: ________________________

Partial Credit

You must show your work on the partial credit problems to receive credit!

13.(12 pts.) Pedestrian A is walking towards the intersection C of two streets intersecting at a right angle. Pedestrian B is walking away from intersection C. Pedestrian A is going North at 2 mph, and Pedestrian B is going East at 3 mph. How fast is the distance from Pedestrian A to Pedestrian B changing when Pedestrian A is 4 miles South of intersection C, and Pedestrian B is 3 miles East of intersection C.



SOLUTION:

Let us denote by $z(t)$ the distance at any time between the pedestrians, by $y(t)$ the distance between A and the intersection and by $x(t)$ the distance between B and the intersection. We are given,

$$\frac{dy}{dt} = -2 \quad ; \quad \frac{dx}{dt} = 3.$$

And we want to know

$$\left.\frac{dz}{dt}\right|_{y=4,x=3}.$$

We know by the Pythagorean Theorem that $z^2(t) = y^2(t) + x^2(t)$ and differentiating with respect to time in both sides gives

$$2z(t)\frac{dz}{dt} = 2y(t)\frac{dy}{dt} + 2x(t)\frac{dx}{dt}$$

$$\Longrightarrow \frac{dz}{dt} = \frac{1}{z(t)}\left[y(t)\frac{dy}{dt} + x(t)\frac{dx}{dt}\right].$$

We also know (again by the Pythagorean theorem) that if $y = 4$ and $x = 3$, then $z = 5$, so that in mph

Name: ______________________________

Instructor: __________________________

$$\left.\frac{dz}{dt}\right|_{y=4,x=3} = \frac{1}{5}\left[4(-2)+3(3)\right] = \frac{1}{5}$$

Name: ____________________________

Instructor: ________________________

14.(12 pts.) Let $C(t)$ be the concentration of a drug in the bloodstream. As the body eliminates the drug, $C(t)$ decreases at a rate that is proportional to the amount of the drug that is present at the time. Thus $C'(t) = kC(t)$, where k is a constant. The initial concentration of the drug is 4 mg/ml. After 5 hours, the concentration is 3 mg/ml.

(a) Give a formula for the concentration of the drug at time t.

(b) How much drug will there be in 10 hours?

(c)How long will it take for the concentration to drop to 0.5 mg/ml?

Solution: (a)

$$C(t) = C(0)e^{kt} = 4e^{kt}$$
$$C(5) = 3 = 4e^{k5} \quad \text{(solve for k)}$$
$$k = \frac{1}{5}\ln\left(\frac{3}{4}\right) \quad \text{(substitute into } C(t)\text{)}$$
$$C(t) = 4\left(\frac{3}{4}\right)^{\frac{1}{5}t}.$$

(b)

$$C(10) = 4\left(\frac{3}{4}\right)^2 = \frac{9}{4}.$$

(c)

$$C(t) = 4\left(\frac{3}{4}\right)^{\frac{1}{5}t} = \frac{1}{2} \quad \text{(solve for t)}$$
$$t = -5\log_{3/4}(8) = \frac{-5\ln 8}{\ln 3 - \ln 4}.$$

Name: ______________________________

Instructor: ___________________________

15. (4 pts.) Please circle "TRUE" if you think the statement is true, and circle "FALSE" if you think the statement is False.

(a)(1 pt. No Partial credit) If $y = \ln x$, $y'' = \dfrac{1}{x^2}$.

FALSE : $y' = 1/x$, $y'' = -1/x^2$

TRUE FALSE

(b)(1 pt. No Partial credit) If $y = \ln(2x)$, $y' = \dfrac{1}{2x}$

FALSE : $y' = 2/2x = 1/x$

TRUE FALSE

(c))(1 pt. No Partial credit) $(e^x)^2 = e^{2x}$.

TRUE by the laws of exponents

TRUE FALSE

(d))(1 pt. No Partial credit) $e^{a+b} = e^a + e^b$ for all real numbers a and b.

FALSE; e.g. $e^2 = e^{1+1} \neq 2e$

TRUE FALSE

Name: ______________________________

Instructor: __________________________

Rough Work

Name: ______________________________

Instructor: ANSWERS

Math 10550, Practice Exam II
March 4, 2026

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 18 pages of the test.
- Each multiple choice question is worth 7 points. Your score will be the sum of the best 10 scores on the multiple choice questions plus your score on questions 13-15.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(•)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(•)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(•)
4.	(a)	(•)	(c)	(d)	(e)
5.	(a)	(b)	(•)	(d)	(e)
6.	(a)	(b)	(c)	(•)	(e)
7.	(a)	(b)	(c)	(d)	(•)
8.	(a)	(•)	(c)	(d)	(e)
9.	(a)	(b)	(•)	(d)	(e)
10.	(a)	(b)	(c)	(•)	(e)
11.	(a)	(b)	(•)	(d)	(e)
12.	(a)	(•)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice ______________

13. ______________

14. ______________

15. ______________

Total ______________